WORK WITH MODELS IN MATHEMATICS AND NATURAL SCIENCES

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The paper focuses on a very important area in mathematics education – use of different kinds of models. Models used in lessons of mathematics are first classified parallelly to models used in natural sciences. Functions of these models are described. Attention is paid to whether and how models in teaching mathematics are used and whether the teachers are able to use the potential they offer. Examples of models are used to illustrate the advantages but also the possible risks of their use.

INTRODUCTION

There are a number of areas in which didactics of mathematics and science education meet. The goal of this paper is to introduce one of these areas, an area that has not attracted much attention yet. It is the area of the use of models and of how pupils understand them. Without any doubt use of models in teaching (not only) mathematics considerably supports both pupils and teachers. However, when we make a decision to use models in a lesson, do we do it in a way that their use does not result in misconceptions or that they do not become a source of obstacles? This is an issue of great importance both for theory and school practice. It becomes even more important with the growing use of computer dynamic models (Jančařík, 2015).

In the paper we will first introduce the use of models and their classification from the point of view of natural sciences and then will show how parallels between these models and mathematical models can be used in didactics of mathematics. We come out of the paradigm that cognitive processes in mathematics and natural sciences are similar in certain respect. In the following section we present examples of models and discuss both their potential as the tool for improving understanding of mathematics by pupils and the dangers the use of these particular models bears.

THEORETICAL BACKGROUND

It is well known that transmission of information with the help of well selected models is very effective. Kuřina (1989) discusses use of schemes, graphs and technical drawings. He states that "much less is known about what role geometrical representations and considerations of geometrical-visual type may play in interpretation of concepts and in problem solving". In our research we consider models in a broader sense; we do not work merely with schemes and drawings but also with physical models. The main advantages Petty (1993) sees in the use of models is that they attract attention, bring change and awaken interest, support conceptualization and are easier to map.

In accordance with Kuřina but in a more general context of more types of models we come out of the belief that a model should considerably contribute to introduction of concepts, description of procedures, problem solving but also to diagnosis and reeducation of misconceptions and to overcoming of obstacles pupils face. The emphasis is put on how pupils approach models and how they work with them.

Computer dynamic models are discussed by Jančařík (2015), who focuses primarily on knowledge that pupils would hardly gain without use of computer models. He characterizes four types of problems in which dynamic models introduce new knowledge that could be hard to visualize using traditional means efficiently. Dynamic models:

- allow us to emphasize elements that are invariant even when parameters change;
- help to discover regularities in elements whose behaviour changes;
- help to discover the situations in which the properties of the model change considerably;
- allow to detect step changes in objects; esp. in some cases a radical change in the model's behaviour points at secondary solutions that are hidden in the place of the sudden change.

MODELS IN NATURAL SCIENCES

Illustrative models have been used in teaching natural sciences for centuries. Information technology has brought a new type of models into lessons – nowadays we can come across not just physical but also virtual models, both two and three dimensional. The more models we use, the greater the need to teach pupils to work with the models is. The ability to work with models and to understand them is not matter-of-course and must be developed (compare Harrison, Treagust, 2000).

The importance of different models and the extent in which they correspond to reality varies greatly. There is a major difference between the model of human skeleton, double helix of DNA and carbon isotope ^{14}C. That is why we distinguish between the following types of models (Jančaříková, 2015):

a) models of common objects,

b) models enabling to see, observe and "touch" objects that are not easily accessible,

c) models asking for abstraction,

d) models representing knowledge from theories.

Models of common objects

These are models of animals, plants and other products of nature, i.e. of objects that pupils may see in vivo (e.g. models of earthworms, ants, blossoms, common mushrooms etc.). Models may be of the same size as the real objects or may differ, may be larger than the observed object (ant, blossom) or smaller in case the real object is too large (e.g. giraffe). If the dimensions of the model are different, it already requires from the pupil to abstract. If we want to develop pupils' ability to *work with models*, we should begin with models that are of equal size as the real object and then move to models requiring abstraction, e.g. the globe.

Models enabling to see, observe and "touch" objects that are not easily accessible

These are models of products of nature, animals and plants or their parts that are common in real world but for various reasons cannot be manipulated with in real life, they are hard to access or inaccessible (to children and pupils). The models enable manipulation impossible in real life. An example of such a model is the model of human body with organs that can be opened to see the organs. Models of this category at the same time allow pupils to look for connections between the model and reality. And this is exactly what should be developed through work with this category of models.

Models requiring abstraction

These are models of objects that are not accessible to all or majority of people, e.g. the above mentioned model of a double helix of DBA or a model of a cell. Models asking for abstraction should be shown to pupils only after sufficient previous work and careful preparation and planning. Use of these models should be preceded by methodologically carefully conceived work with models from the previous category.

Models representing knowledge from theories

A specific category are models representing the latest scientific, not yet empirically verified theories. An example of this might be e.g. the historically changing model of an atom or models of solar system. Scientists have used and use models of this category to verify and improve their theories about how the observed phenomenon works.

CONTENT TRANSFORMATION

Understanding that some model or picture represents an object or a product of nature is a process that is called content transformation, or **isomorphism** (Slavík et al., 2013). This concept is also used by linguists to express that words, sentences and stories describe real life. The concept of isomorphism expresses the relationship between an image and real objects, people, animals, plants or landscapes.

Before we use a model of an object that pupils cannot see in real life and have no experience with, they must have worked with models of objects they are familiar with, that they can see and touch with their own hands.

MODELS IN MATHEMATICS

When working with models in mathematics one must be aware of the fact that a vast majority of models belongs to the category of models requiring abstraction. Concepts used in mathematics are not of real physical nature. In real life we cannot find a point, line segment, straight line or numbers. Using models and drawings we teach pupils to abstract and generalize in their minds the essence of the separate models in such a way that enables them to get an insight into the abstract world of mathematics and to work there with ideal concepts (Vopěnka, 2011). That is why we have to put a lot of emphasis on making sure that models we present to pupils have as much of the represented object as possible and to represent it as fully as possible. If these two criteria are ignored, the outcome might be the creation of a non-model in pupils' minds on the one hand (the object does not have the required properties) or an incomplete model on the other hand (it has all the desired properties but is limited by other conditions).

Transformation of content in mathematics

Models in mathematics do not correspond to the objects they represent. When transforming the content, pupils must change the substance of the transformed concept from physical to abstract. It may even happen that the model has different properties than the object it represents. An example of this may be a model of a triangle whose sides are made of wire. The centre of gravity of this model will be somewhere else than in case of a model created from paper or metal plate. This means the mathematical concept of the centre of gravity gets separated from the same concept in physics. Similarly infinite objects we work with (for example straight lines, half rays or planes) are not infinite when represented by models. Many pupils find it very difficult to switch between models and objects they represent. It may be for example extremely difficult to visualize intersection of skew lines outside the area of the model. Analogically to natural sciences, in mathematics pupils have to learn how to approach models, how to make use of their properties and must be aware of the limits of these models.

CONCLUSION

Models are generally considered to be a tool that supports conceptualization and easier remembering of concepts. However, understanding the relationship between models and real objects is not matter-of-course. Mere experimenting with models will not make pupils understand the differences between mathematical and real worlds. This must be facilitated by the teacher. If a model is used in a wrong way, its use may result in pupils' misconceptions. For example pupils must learn that while making a geometrical construction they cannot work with thickness of the line, size of a point or the third dimension of the model of two-dimensional figures. The here presented papers points at some of the dangers attached to the use of models and shows that a lot of attention must be paid while introducing them to teaching both in mathematics and natural sciences.

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